Pymble Ladies' College

Mathematics Advanced
HSC Trial Examination
Term 3 2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

Total marks 100

Section 1 – 10 marks (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-35)

- Attempt Questions 11-34
- Allow about 2 hours and 45 minutes for this section

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Name:.....

Student Number:

Teacher's Name:.....

Mathematics Advanced
HSC Trial Examination

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

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1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

6 A ○ B ○ C ○ D ○

7 A ○ B ○ C ○ D ○

8 A ○ B ○ C ○ D ○

9 A ○ B ○ C ○ D ○

10 A ○ B ○ C ○ D ○

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SECTION I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

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1 The table below shows the number of students that make up a school's debating team.

	Junior	Senior
Male	4	6
Female	10	5

A student is picked at random from the debating team. Given that the student is female, what is the probability the student is a junior?

(A) $\frac{2}{3}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{3}$

2 Which of the following is equivalent to $\frac{2^n - 2^{n-1}}{2^{n+1} + 2^n}$?

(A) $-\frac{1}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{6}$

(D) $\frac{1}{2}$

3 A sector with perimeter 28 cm contains an angle of 1.5 radians at the centre of a circle with radius r cm. What is the value of r?

(A) 6

(B) 8

(C) 10

(D) 12

4 The marks on a test are normally distributed with mean μ and standard deviation σ.
2.5% of the marks are 34 or less and 16% of the marks are 70 or more.
What is the value of μ?

(A) 54

(B) 56

(C) 58

(D) 60

5 What is the domain of the function $f(x) = \ln(x-1) + \sqrt{2-x}$?

(A) $x \in (1, 2)$

(B) $x \in (1, 2]$

(C) $x \in [1, 2)$

(D) $x \in [1, 2]$

6 Nidhi owns equipment that is used for 10 hours per day for all 365 days of the year.
The value of the equipment is depreciated by Nidhi using the unit cost method.
The value of the equipment E_n, in dollars, after n years can be modelled by the recurrence relation

$$E_0 = 100\,000, \quad E_{n+1} = E_n - 5475$$

By what value is the equipment depreciated?

(A) \$1.50 per hour

(B) \$10 per hour

(C) \$15 per hour

(D) \$1.50 per day

7 Which of the following is an expression for $\int\left(\frac{1}{1+\sin x}+\frac{1}{1-\sin x}\right)dx$?

(A) $2\sec^2 x+c$

(B) $4\tan x\sec^2 x+c$

(C) $2\tan x+c$

(D) $2x+2\sec x+c$

8 The function $y=\log_{10} x$ is transformed to $y=3\log_{10}\left[2(x+5)\right]-4$ using only four steps. Which of the following is **not** one of the correct steps to transform $y=\log_{10} x$ to the new graph?

(A) Translate down by 4 units

(B) Translate to the left by 5 units

(C) Vertical dilation by a factor of 3

(D) Horizontal dilation by a factor of 2

9 Which of the following is an expression for $\frac{d}{dx}\left(7^x\right)$?

(A) $\frac{1}{7x\log_e 7}$

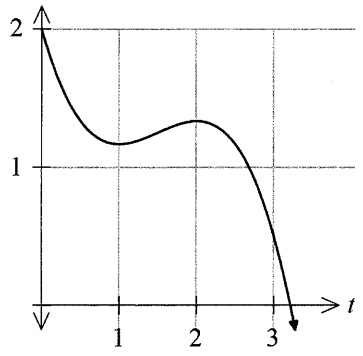
(B) $\frac{1}{x\log_e 7}$

(C) $\frac{\log_e 7}{7}$

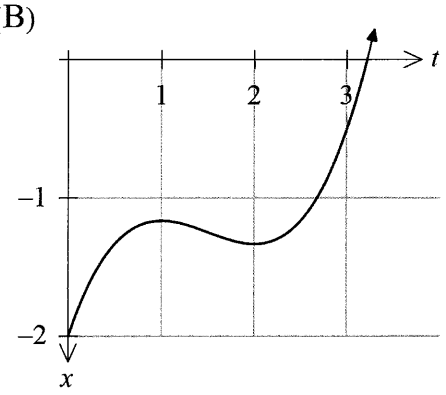
(D) $7^x\times\log_e 7$

10 A particle P moves along a straight line such that its acceleration with respect to time is given by $a = -2t + 3$, where a is the acceleration of the particle at time t seconds for $t \geq 0$.
The particle is at rest when $t = 1$ and $t = 2$.
Which of the following graphs best represents the displacement of the particle?

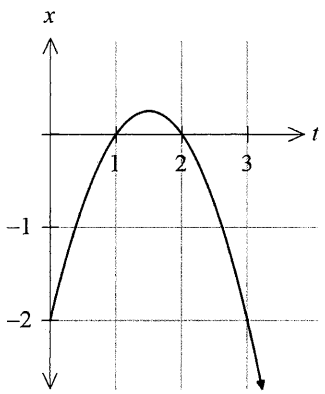
(A)



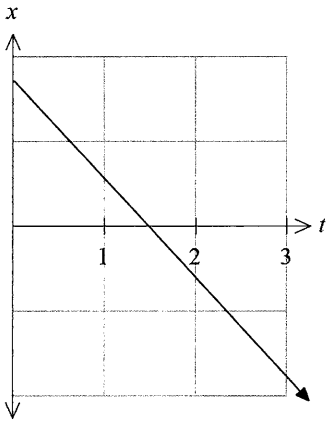
(B)



(C)



(D)



End of Section I

Name.....

Student Number:

Teacher's Name.....

Mathematics Advanced
Section II

Answer Booklet

90 marks
Attempt Questions 11-34
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
- Extra writing space is provided at the end of the paper. If you use the additional space, clearly indicate which question you are answering.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Solve

$$\log_3(12y+5)-\log_3(1-3y)=2$$ **3**

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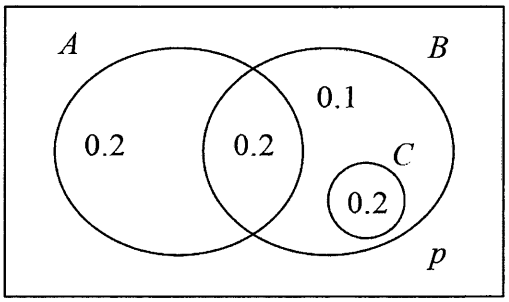
Question 12 (2 marks)



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p. **1**

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(b) Write down a pair of mutually exclusive events from A, B and C. **1**

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Question 13 (2 marks)

Find

$\int \frac{3x^4 - 4}{2x^3}\, dx$ writing your answer in simplest form. **2**

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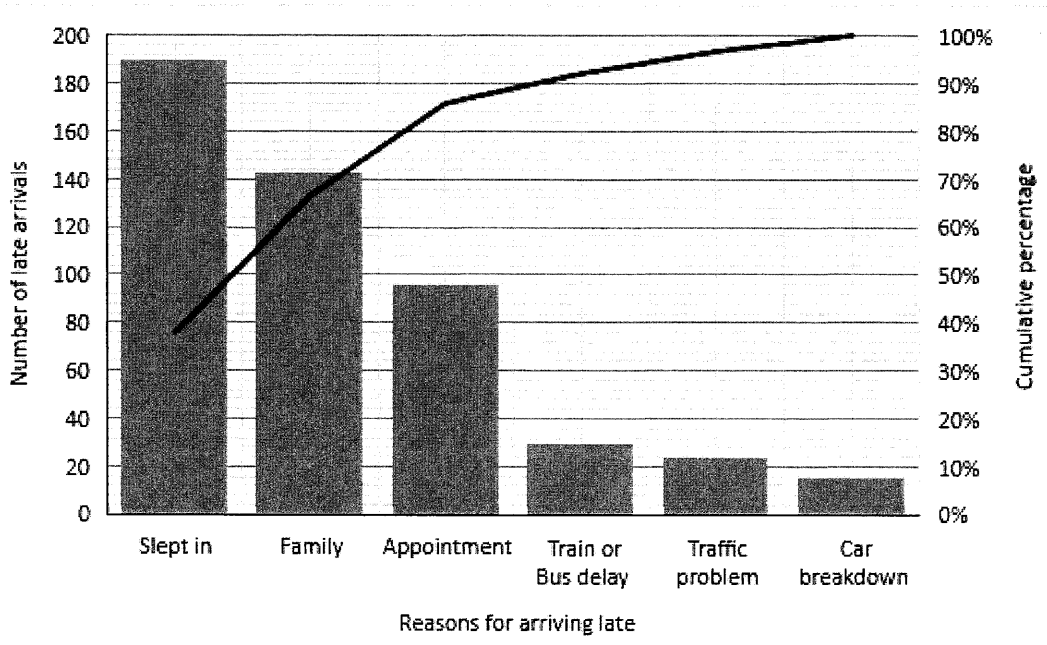
Question 14 (1 mark)

A school collected data related to the reasons given by students arriving late. **1**
The pareto chart shows the data collected.



What percentage of students gave the reason "Train or Bus delay"?

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Question 15 (6 marks)

The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C. **2**

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(ii) the exact radius of C, giving your answer as a simplified surd. **1**

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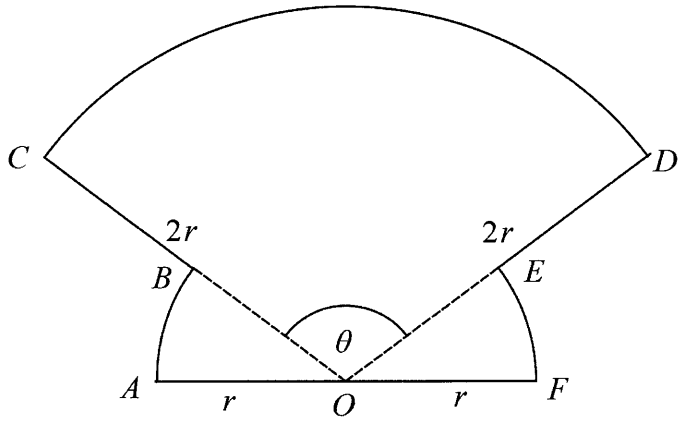
Question 15 continues on next page

Question 15 continued.

The line l has equation $y = 3x + k$ where k is a constant.
Given that l is a tangent to C,

(b) find the possible values of k, giving you answers as simplified surds. **3**

Question 16 (5 marks)



NOT TO SCALE

Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.
In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*. **1**

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Question 16 continues on next page

Question 16 continued.

(b) show that the area of the logo is **2**

$$\frac{1}{2}r^2(3\theta+\pi).$$

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(c) find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π. **2**

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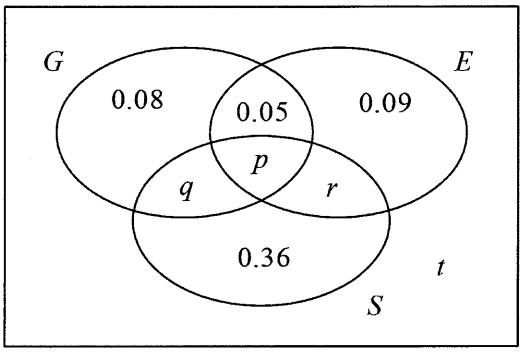
Question 17 (8 marks)

A large College produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the College is selected at random and the events G, E and S are defined as follows:

G is the event that the student reads the magazine about green issues.
E is the event that the student reads the magazine about equality.
S is the event that the student reads the magazine about sports.

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines. **1**

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No students read all three magazines and $\mathrm{P}(G) = 0.25$.

(b) Find

(i) the value of p. **1**

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(ii) the value of q. **1**

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Question 17 continues on next page

Question 17 continued.

Given that $P(S \mid E) = \frac{5}{12}$

(c) Find

(i) the value of r. **2**

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(ii) the value of t. **1**

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(d) Determine whether or not the events $(S \cap E')$ and G are independent. **2**
Show your working clearly.

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Question 18 (2 marks)

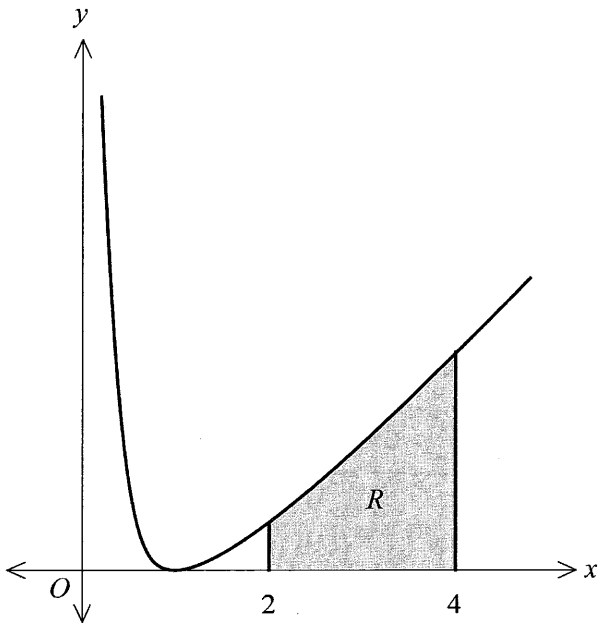

Figure 2

Figure 2 shows a sketch of part of the curve with equation **2**

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

Use the trapezoidal rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

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Question 19 (2 marks)

A student's attempt to solve the question **2**

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

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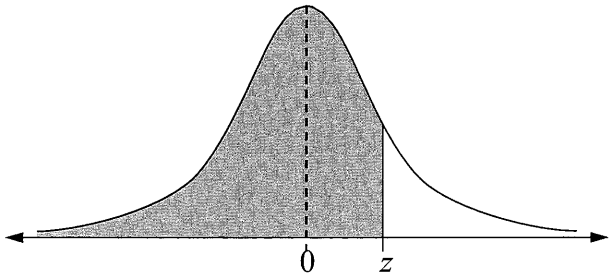
You may use the information below to answer Question 20

Table of values $P(Z \leq z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

Question 20 (3 marks)

The heights of females from Mathimagic land are normally distributed with **3**

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1.02% of females from Mathimagic land are shorter than k cm, use the table provided to find the value of k.

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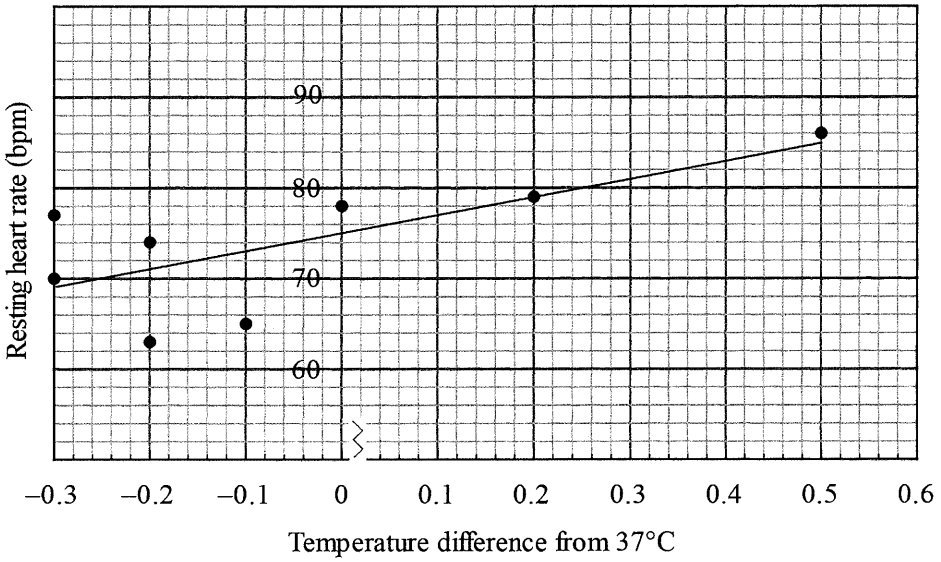
Question 21 (4 marks)

A healthy human body temperature is 37.0°C.
Eight randomly selected people were examined by medical staff. The difference in their body temperature from 37.0°C (in degrees) and resting heart rate (in beats per minute) were recorded.

Temperature difference from 37.0°C (x)	−0.2	−0.3	−0.3	−0.2	−0.1	0	0.2	0.5
Heart rate (y)	63	77	70	74	65	78	79	86

These results are displayed on the graph below.



(a) The least-squares regression line has been plotted on the graph. Find the equation of this line. **2**

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(b) By using the equation of the regression line, predict the resting heart rate of a person with a body temperature of 37.4°C. **1**

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(c) Explain why the least-squares regression line would not be reliable to predict the resting heart rate of a person with a body temperature of 37.6°C? **1**

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Question 22 (4 marks)

The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started. **1**

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On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures. **2**

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On 1st January 2020, the owners of the nature reserve announced a long term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan. **1**

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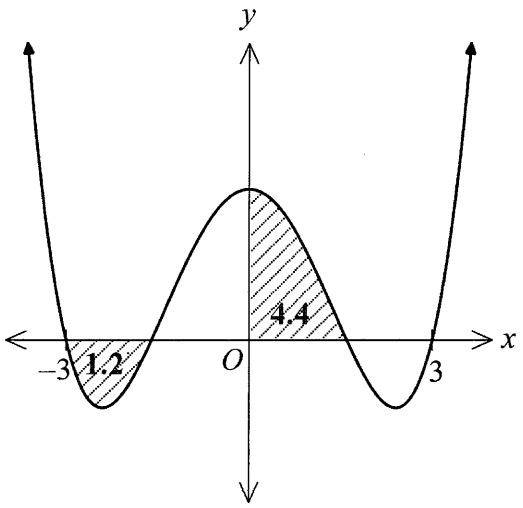
Question 23 (1 mark)

The graph of the function $y = f(x)$ below is symmetrical about the y-axis. The areas of the shaded regions are given.



Find $\int_{-3}^{3} f(x)\,dx$. **1**

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Question 24 (3 marks)

Prove that $\dfrac{\sec x \tan x + \sec^2 x}{(\tan x + \sec x)^2 + 1} = \dfrac{1}{2}$. **3**

Question 25 (2 marks)

Determine the values of x for which the graph of $y = 3\cos\left(\frac{5x-1}{2}\right)$ crosses the x-axis in the interval $0 \leq x \leq \pi$. **2**

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Question 26 (2 marks)

Find the value of k, given that $\int_1^k \frac{2}{\sqrt{x}}\,dx = 8$. **2**

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Question 27 (6 marks)

Hen Solo made a profit of $20 000 in its first year of trading with her free-range egg farm. A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.
According to the model,

(a) show that the profit for Year 3 will be $23 328. **1**

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(b) find the first year when the yearly profit will exceed $65 000. **3**

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(c) find the total profit for the first 20 years of trading, giving your answer to the nearest $1000. **2**

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Question 28 (7 marks)

A continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases}0 & x<1\\ \dfrac{2}{x^2} & 1\le x\le 2\\ 0 & x>2\end{cases}$$

(a) Show that $f(x)$ is a probability density function. **2**

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(b) Find the cumulative distribution function $F(x)$ of the random variable X. **2**

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Question 28 continues on next page

Question 28 continued

(c) Show that if Q_1, Q_2, Q_3 are the quartiles of the distribution of X, then **3**

$\frac{1}{Q_1}, \frac{1}{Q_2}, \frac{1}{Q_3}$ are consecutive terms in an arithmetic sequence.

Question 29 (5 marks)

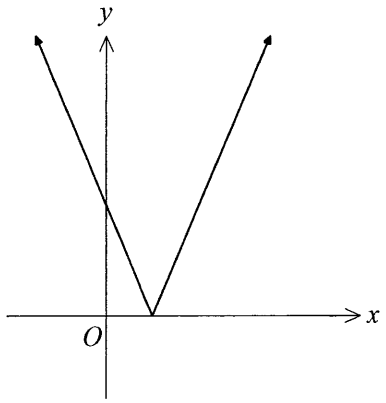
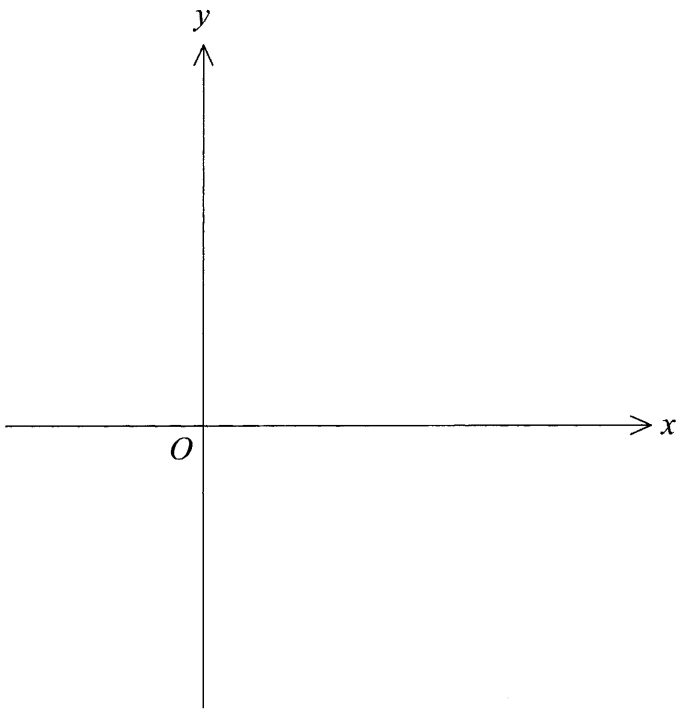


Figure 3

Figure 3 shows a sketch of the graph with equation $y = |2x - 3k|$ where k is a positive constant.

(a) Sketch the graph with equation $y = h(x)$ where $h(x) = k - |2x - 3k|$ stating **3**

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes



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Question 29 continues on next page

Question 29 continued.

(b) Find, in terms of k, the coordinates of the minimum point of the graph with equation **2**

$$y = 3 - 5h\left(\frac{1}{2}x\right)$$

Question 30 (5 marks)

The velocity of a particle moving along the x-axis is given by

$$V = 10 - 10e^{-2t}$$

where t is the time in minutes and x is the displacement in metres.

(a) Show that the particle is initially at rest. **1**

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(b) Show that the acceleration of the particle is always positive. **1**

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(c) Explain why the particle is moving in a positive direction for all $t > 0$. **1**

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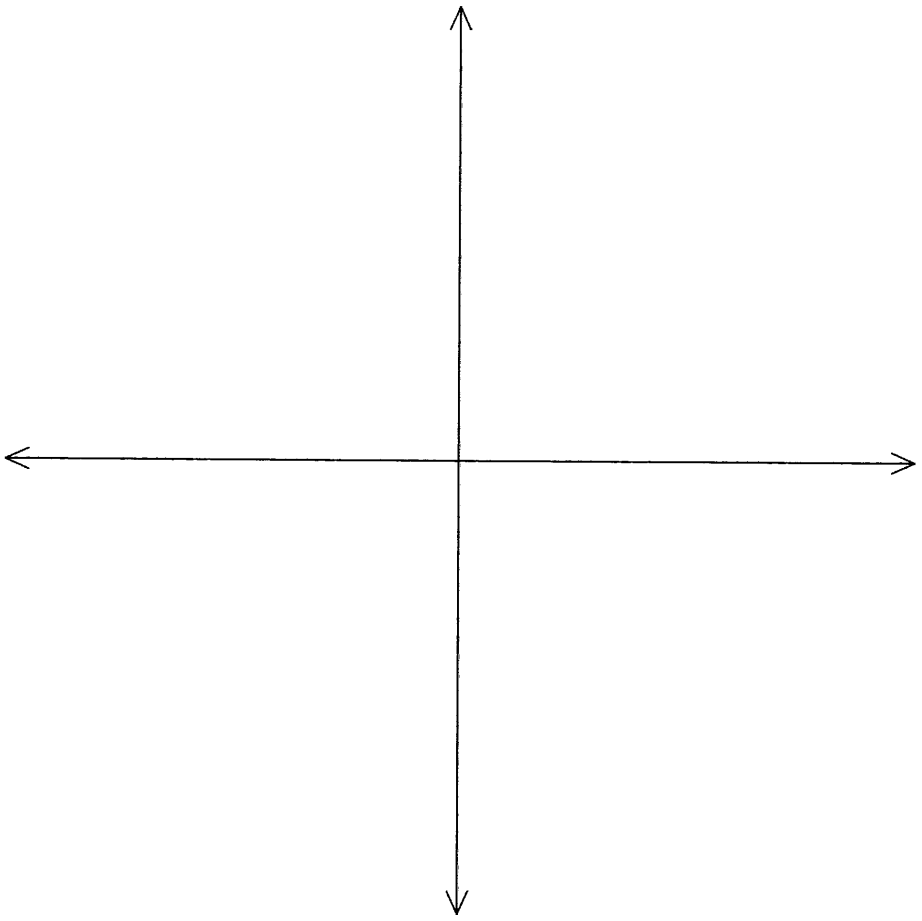
Question 30 continues on next page

Question 30 continued.

(d) Sketch the graph of the particle's velocity as a function of time. **2**



Question 31 (4 marks)

(a) Differentiate $3+\sin 2x$ **1**

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(b) Hence, or otherwise, find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 2x}{3+\sin 2x}\,dx$. **3**

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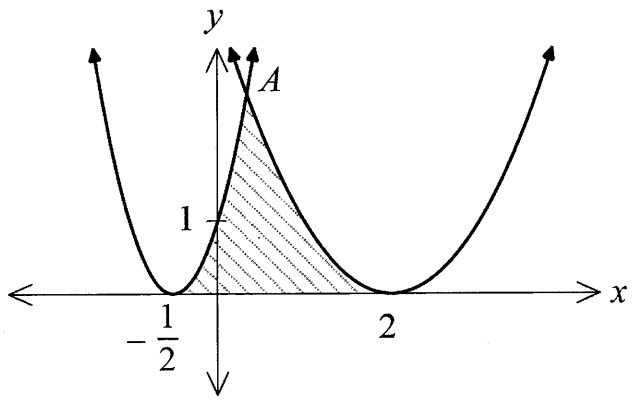
Question 32 (3 marks)

Given that $y = \dfrac{x-4}{2+\sqrt{x}}$, $x > 0$ find the value of A such that $\dfrac{dy}{dx} = \dfrac{1}{A\sqrt{x}}$. **3**

Question 33 (4 marks)

Two curves, $y=(2x+1)^2$ and $y=(x-2)^2$, intersect as shown in the diagram below.



(a) Show that A has x-coordinate $\frac{1}{3}$. **1**

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(b) The region bounded by $y=(2x+1)^2$, $y=(x-2)^2$ and the x-axis is shaded. **3**
Find the exact area of the shaded region.

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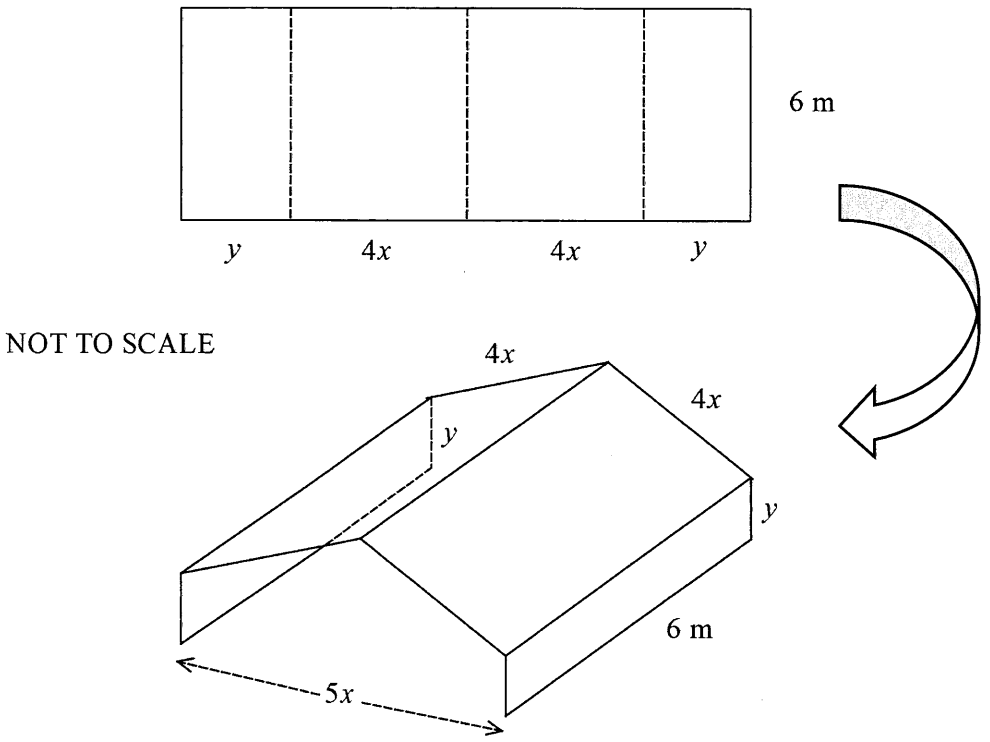
Question 34 (6 marks)

A rectangular canvas sheet is folded to form the cover of a tent with a pentagonal cross-section, as shown in the following diagrams.



The vertical heights on the sides of the tent are y metres and the slant lengths are $4x$ metres.
The horizontal distance between the vertical sides is $5x$ metres and the width of the tent is 6 metres.

(a) Show that $y = 10 - 4x$, given the area of the canvas sheet is 120 m^2. **1**

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Question 34 continues on next page

Question 34 continued.

The following diagram shows the cross-section of the tent, where h is the height of the top of the tent from the vertical sides.

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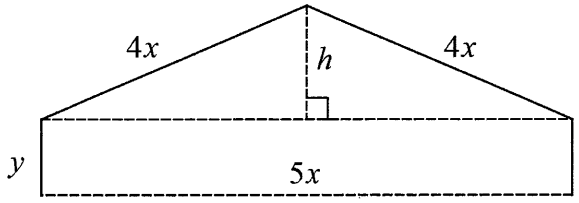

(b) Show that $h = \frac{\sqrt{39}x}{2}$. **2**

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Question 34 continues on next page

Question 34 continued.

(c) Find the exact value of x which maximises the volume of the tent. **3**

END OF PAPER

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Pymble Ladies' College

Mathematics Advanced
HSC Trial Examination
Term 3 2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

Total marks 100

Section 1 – 10 marks (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-35)

- Attempt Questions 11-34
- Allow about 2 hours and 45 minutes for this section

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Name:.....

Student Number:

Teacher's Name:.....

Mathematics Advanced
HSC Trial Examination

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct ↙

A ⊗ B ⊗ C ○ D ○

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1 A ● B ○ C ○ D ○

2 A ○ B ○ C ● D ○

3 A ○ B ● C ○ D ○

4 A ○ B ○ C ● D ○

5 A ○ B ● C ○ D ○

6 A ● B ○ C ○ D ○

7 A ○ B ○ C ● D ○

8 A ○ B ○ C ○ D ●

9 A ○ B ○ C ○ D ●

10 A ● B ○ C ○ D ○

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SECTION I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

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1 The table below shows the number of students that make up a school's debating team.

	Junior	Senior
Male	4	6
Female	10	5

A student is picked at random from the debating team. Given that the student is female, what is the probability the student is a junior?

(A) $\frac{2}{3}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{3}$

$\frac{10}{15} = \frac{2}{3}$

2 Which of the following is equivalent to $\frac{2^n - 2^{n-1}}{2^{n+1} + 2^n}$?

(A) $-\frac{1}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{6}$

(D) $\frac{1}{2}$

$\frac{2^{n-1}(2-1)}{2^n(2+1)}$

$= \frac{1}{2 \times 3}$

$= \frac{1}{6}$

3 A sector with perimeter 28 cm contains an angle of 1.5 radians at the centre of a circle with radius r cm. What is the value of r?

(A) 6

(B) 8

(C) 10

(D) 12

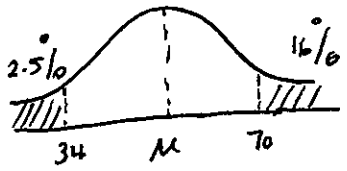
$28 = 2r + r \times 1.5$

$3.5r = 28$

$r = \frac{28}{3.5}$

$= 8$

4 The marks on a test are normally distributed with mean μ and standard deviation σ. 2.5% of the marks are 34 or less and 16% of the marks are 70 or more. What is the value of μ?



(A) 54

(B) 56

(C) 58

(D) 60

$\mu - 2\sigma = 34$

$2\mu + 2\sigma = 140$

$3\mu = 174$

$\mu = 58$

5 What is the domain of the function $f(x) = \ln(x-1) + \sqrt{2-x}$?

(A) $x \in (1, 2)$

(B) $x \in (1, 2]$

(C) $x \in [1, 2)$

(D) $x \in [1, 2]$

$x > 1 \quad x \leq 2$

$\therefore x \in (1, 2]$

6 Nidhi owns equipment that is used for 10 hours per day for all 365 days of the year. The value of the equipment is depreciated by Nidhi using the unit cost method. The value of the equipment E_n, in dollars, after n years can be modelled by the recurrence relation

$$E_0 = 100\,000, \quad E_{n+1} = E_n - 5475$$

By what value is the equipment depreciated?

(A) \$1.50 per hour

(B) \$10 per hour

(C) \$15 per hour

(D) \$1.50 per day

10 hr/day 365 days

$E_1 = 100000 - 5475$

$\text{Depreciation} = \dfrac{5475}{365 \times 10}$

$= \$1.50/\text{hr}$

7 Which of the following is an expression for $\int\left(\frac{1}{1+\sin x}+\frac{1}{1-\sin x}\right)dx$?

(A) $2\sec^2 x+c$

(B) $4\tan x\sec^2 x+c$

(C) $2\tan x+c$

(D) $2x+2\sec x+c$

$\int\left(\frac{1-\sin x+1+\sin x}{1-\sin^2 x}\right)dx$

$=\int\frac{2}{\cos^2 x}\,dx$

$=2\int\sec^2 x\,dx = 2\tan x+C$

8 The function $y=\log_{10}x$ is transformed to $y=3\log_{10}\left[2(x+5)\right]-4$ using only four steps. Which of the following is **not** one of the correct steps to transform $y=\log_{10}x$ to the new graph?

(A) Translate down by 4 units

(B) Translate to the left by 5 units

(C) Vertical dilation by a factor of 3

(D) Horizontal dilation by a factor of 2

9 Which of the following is an expression for $\frac{d}{dx}\left(7^x\right)$?

(A) $\frac{1}{7x\log_e 7}$

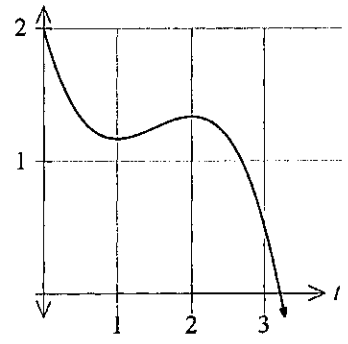
(B) $\frac{1}{x\log_e 7}$

(C) $\frac{\log_e 7}{7}$

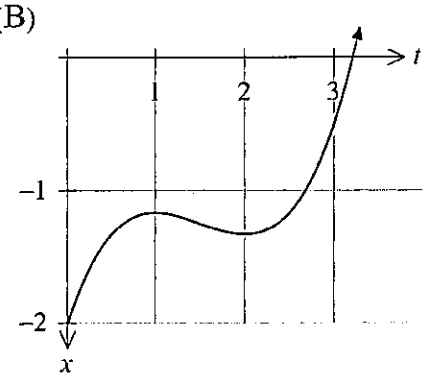
(D) $7^x\times\log_e 7$

10 A particle P moves along a straight line such that its acceleration with respect to time is given by $a = -2t + 3$, where a is the acceleration of the particle at time t seconds for $t \geq 0$.
The particle is at rest when $t = 1$ and $t = 2$.
Which of the following graphs best represents the displacement of the particle?

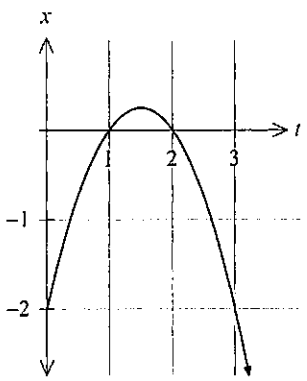
(A)



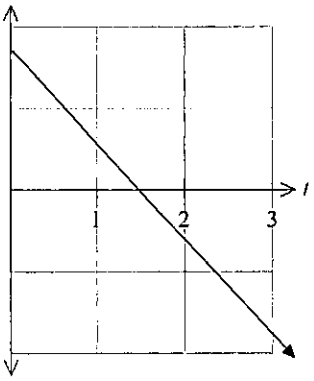
(B)



(C)



(D)



$a = -2t + 3$

$v = \int (-2t + 3)\, dt$

$v = -t^2 + 3t + c_1$

when $t = 1$ $v = 0$

$\therefore \quad 0 = -1 + 3 + c_1$

$\therefore \quad c_1 = -2$

$v = -t^2 + 3t - 2$

$x = \int (-t^2 + 3t - 2)\, dt$

$= -\frac{t^3}{3} + \frac{3t^2}{2} - 2t + c_2$

End of Section I

Name.....

Student Number:

Teacher's Name.....

Mathematics Advanced
Section II

Answer Booklet

90 marks
Attempt Questions 11-34
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
- Extra writing space is provided at the end of the paper. If you use the additional space, clearly indicate which question you are answering.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Solve

$$\log_3(12y+5)-\log_3(1-3y)=2$$ 3

$\log_3 \frac{12y+5}{1-3y} = 2$

$\therefore \frac{12y+5}{1-3y} = 3^2$

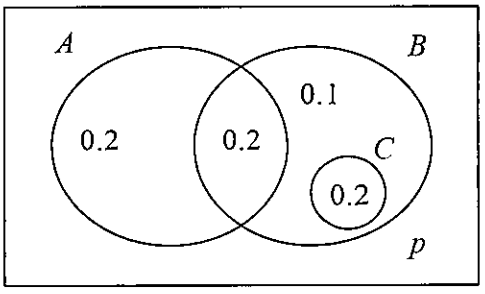
$12y+5 = 9(1-3y)$

$39y = 4$

$y = 4/39$

3	Correct
2	Correctly removing log and progress towards solving eqn
1	Just removing the log correctly.

Question 12 (2 marks)



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p. 1

$p = 1-(0.2+0.2+0.1+0.2)$ 1 c/w

$= 0.3$

(b) Write down a pair of mutually exclusive events from A, B and C. 1

A and C 1 c/w

Question 13 (2 marks)

Find

$\int \frac{3x^4 - 4}{2x^3}\,dx$ writing your answer in simplest form. **2**

$= \frac{3}{2}\int x\,dx - 2\int x^{-3}\,dx$

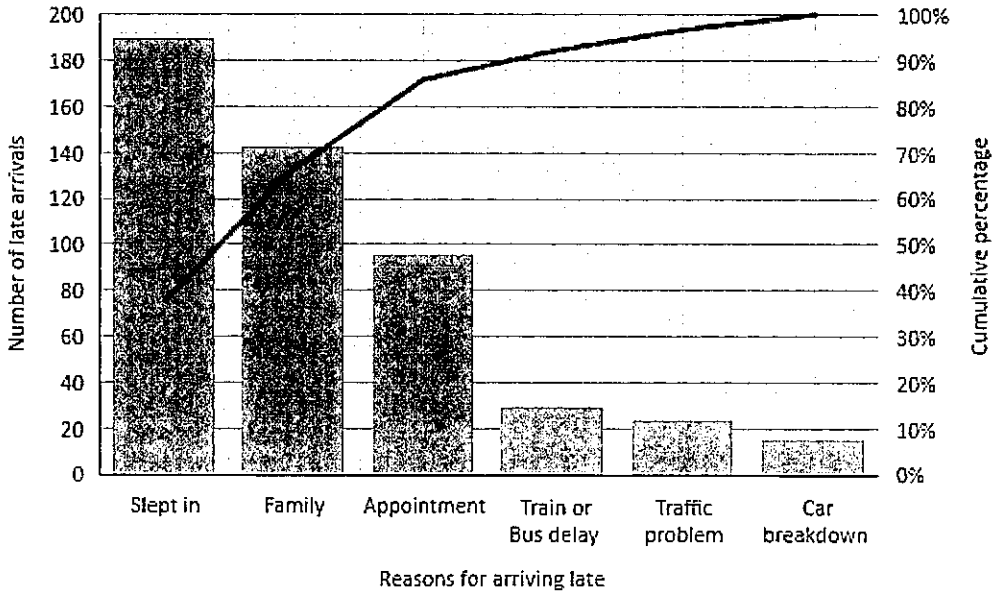
$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{-2} + C$

$= \frac{3}{4}x^2 + \frac{1}{x^2} + C$

2	correct
1	correctly separating integral and some progress

Question 14 (1 mark)

A school collected data related to the reasons given by students arriving late. **1**
The pareto chart shows the data collected.



What percentage of students gave the reason "Train or Bus delay"?

$92 - 86 = 6\%$ 1 r/w

Question 15 (6 marks)

The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C. 2

$x^2 - 10x + 25 + y^2 + 4y + 4 = -11 + 25 + 4$

$(x-5)^2 + (y+2)^2 = 18$ — 2 | Correct

— 1 | Correcting

Centre: $(5, -2)$ | Completing the Square on either x or y

(ii) the exact radius of C, giving your answer as a simplified surd. 1

$r = \sqrt{18}$ ✓1 ω

$= 3\sqrt{2}$ (Accept $\sqrt{18}$)

Question 15 continues on next page

Question 15 continued.

The line l has equation $y = 3x + k$ where k is a constant.
Given that l is a tangent to C,

(b) find the possible values of k, giving you answers as simplified surds. **3**

$(x-5)^2 + (y+2)^2 = 18$

$y = 3x + k$

Solving simultaneously

$(x-5)^2 + (3x+k+2)^2 = 18$

$x^2 - 10x + 25 + 9x^2 + 3xk + 6x + 3xk + k^2 + 2k + 6x + 2k + 4 = 18$

$10x^2 + 2x + 6xk + k^2 + 4k + 11 = 0$

For line to be a tangent, there must be only one solution $\therefore \Delta = 0$

$\Delta = (2+6k)^2 - 4 \times 10(k^2 + 4k + 11) = 0$

$4 + 24k + 36k^2 - 40k^2 - 160k - 440 = 0$

$-4k^2 - 136k - 436 = 0$

$k^2 + 34k + 109 = 0$

$k = \dfrac{-34 \pm \sqrt{34^2 - 4 \times 109}}{2}$

$k = -17 \pm 6\sqrt{5}$

3	Correct
2.	Solving simultaneously or finding $\frac{dy}{dx} = 3$ Deducing $\Delta = 0$ Some progress
1.	Just substituting y and simplifying. or rearranging the equation and find $y = \pm$ ___

Question 16 (5 marks)

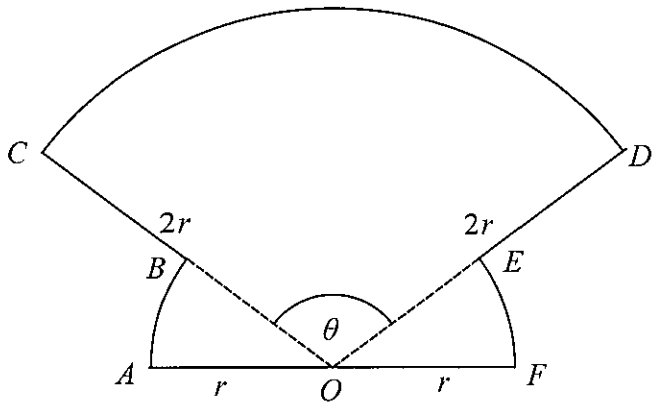


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Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.
In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*. **1**

$\angle AOB = \frac{\pi - \theta}{2}$ 1/r/w

Angle must be in radians

Question 16 continues on next page

Question 16 continued.

(b) show that the area of the logo is 2

$$\frac{1}{2}r^2(3\theta+\pi).$$

Area of sector AOB $= \frac{1}{2} \times r^2\left(\frac{\pi-\theta}{2}\right)$

Area of sector OEF $= \frac{r^2}{2}\left(\frac{\pi-\theta}{2}\right)$ also

Area of sector COD $= \frac{1}{2} \times (2r)^2\theta$

$= 2r^2\theta$

Total area $= 2 \times \frac{r^2}{2}\left(\frac{\pi-\theta}{2}\right) + 2r^2\theta$

$= \frac{r^2}{2}(\pi-\theta) + 2r^2\theta$

$= \frac{r^2}{2}(\pi-\theta+4\theta)$

$= \frac{r^2}{2}(\pi+3\theta)$ As req'd

2 | Correct

1 | At least two Component areas Correct. i.e COD and AOB or OEF NOT AOB and OEF only

(c) find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π. 2

Perimeter $= 2r + 2 \times r\left(\frac{\pi-\theta}{2}\right) + 2r + 2r \times \theta$

$= 4r + r\pi - r\theta + 2r\theta$

$= 4r + r\pi + r\theta$

2 | Correct

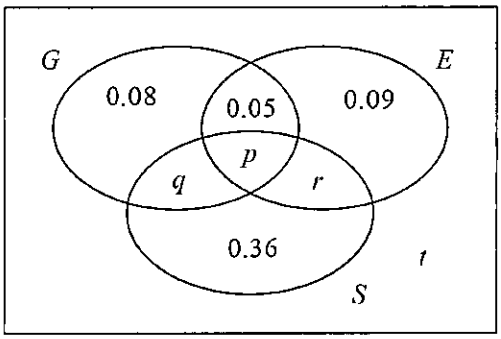
1 | Some progress

Question 17 (8 marks)

A large College produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the College is selected at random and the events G, E and S are defined as follows:

G is the event that the student reads the magazine about green issues.
E is the event that the student reads the magazine about equality.
S is the event that the student reads the magazine about sports.

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines. **1**

Proportion who read 1 magazine 1 r/w
$= 0.08 + 0.09 + 0.36$
$= 0.53$

No students read all three magazines and $\mathrm{P}(G) = 0.25$.

(b) Find

(i) the value of p. **1**

$p = 0$ since no students read all 3 magazines 1 r/w

(ii) the value of q. **1**

$q + 0.05 + 0.08 = 0.25$ 1 r/w
$q = 0.12$

Question 17 continues on next page

Question 17 continued.

Given that $P(S \mid E) = \frac{5}{12}$

(c) Find

(i) the value of r. 2

$P(S \mid E) = \frac{P(S \cap E)}{P(E)}$

$= \frac{5}{12}$

$\therefore \frac{r}{0.14 + r} = \frac{5}{12}$; $p = 0$

$12r = 0.7 + 5r$

$7r = 0.7$

$r = 0.1$

2	correct
1	progress w. correct eqn.

(ii) the value of t. 1

$t = 1 - (0.08 + 0.05 + 0.09 + 0.36 + 0.12 + 0.1)$

$= 0.2$

1 r/w

(d) Determine whether or not the events $(S \cap E')$ and G are independent. 2

Show your working clearly.

$S \cap E' = 0.36 + q = 0.48$

$(S \cap E') \cap G = q = 0.12$ $P(G) = 0.25$

* $P(S \cap E') \times P(G) = 0.48 \times 0.25 = 0.12$

* $P((S \cap E') \cap G) = 0.12$

Since $P(S \cap E') \times P(G) = P((S \cap E') \cap G)$

the events are independent

2	correct
1	Correctly Calculating either *

Question 18 (2 marks)

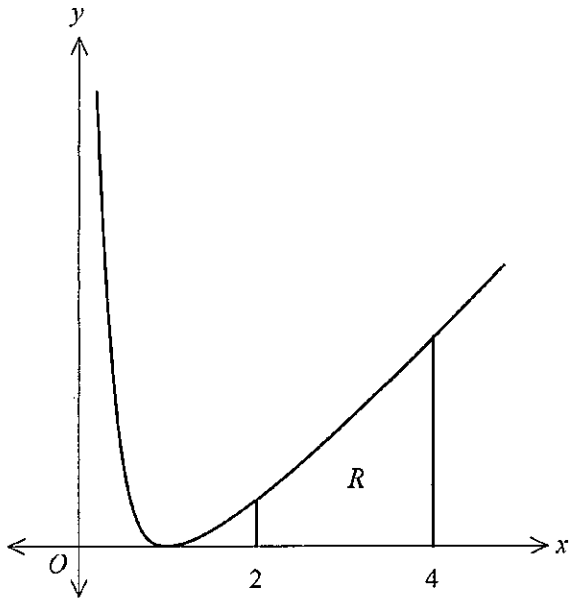


Figure 2

Figure 2 shows a sketch of part of the curve with equation **2**

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

Use the trapezoidal rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

Area $= \frac{0.5}{2}(0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694))$

$= 2.408525$ 2 correct

$\doteq 2.41$ units^2 (3 sig fig) 1 Error in Calculation

Question 19 (2 marks)

A student's attempt to solve the question **2**

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

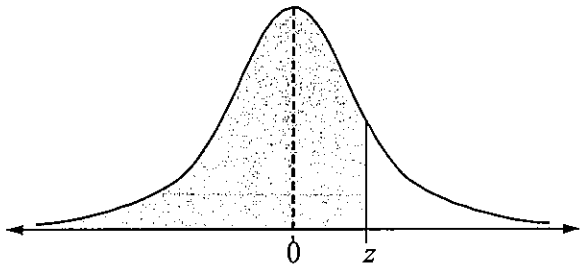
Error 1: From line 3 to 4, dividing away sin x thus eliminating a solution

Error 2: From line 5 to 6, not looking at the domain and providing only one solution.

2	Both errors
1	Only one correct error

You may use the information below to answer Question 20

Table of values $P(Z \leq z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

Question 20 (3 marks)

The heights of females from Mathimagic land are normally distributed with 3

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

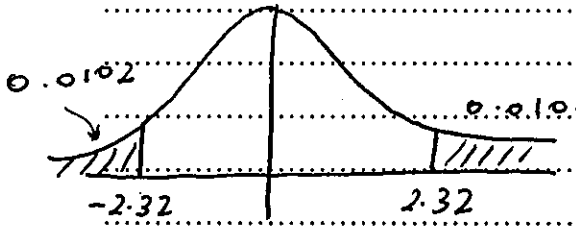
Given that 1.02% of females from Mathimagic land are shorter than k cm, use the table provided to find the value of k.

$1 - 0.0102 = 0.9898$

From table, $0.9898 \rightarrow z = 2.32$

$\frac{k - 166.5}{6.1} = -2.32$

$k = 152.348$ cm



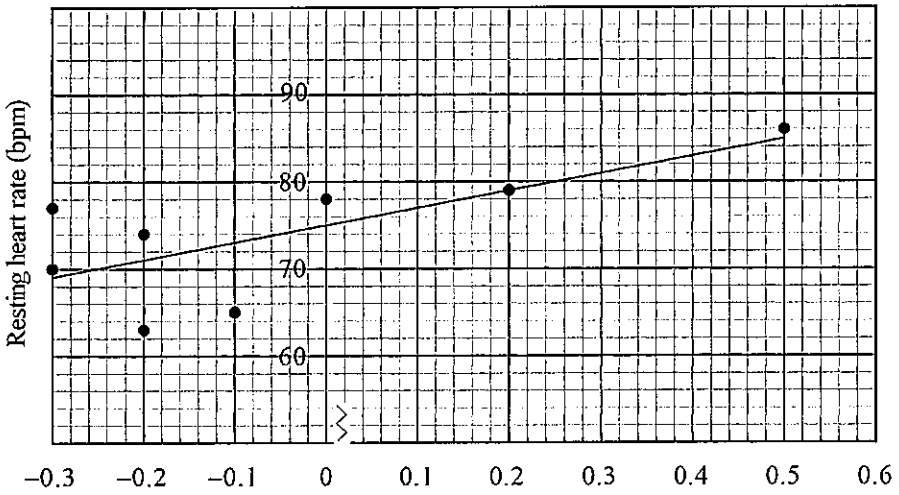
3	Correct
2	Determines actual z-score should be -2.32 OR calculates k associated with 2.32
1	Correct interpretation of the given information via a diagram. OR Finds relevant z-score of 2.32 from table

Question 21 (4 marks)

A healthy human body temperature is 37.0°C.
Eight randomly selected people were examined by medical staff. The difference in their body temperature from 37.0°C (in degrees) and resting heart rate (in beats per minute) were recorded.

Temperature difference from 37.0°C (x)	−0.2	−0.3	−0.3	−0.2	−0.1	0	0.2	0.5
Heart rate (y)	63	77	70	74	65	78	79	86

These results are displayed on the graph below.



use regression stats on calculator

(a) The least-squares regression line has been plotted on the graph. Find the equation of this line. 2

$y = 74.96 + 19.26x$

2 | correct

1 | one piece of the eqn correct OR uses 2 points to find equation of line

(b) By using the equation of the regression line, predict the resting heart rate of a person with a body temperature of 37.4°C. 1

$y = 74.96 + 19.26(0.4)$ 1 r/w

$= 82.664$ beats/min

(c) Explain why the least-squares regression line would not be reliable to predict the resting heart rate of a person with a body temperature of 37.6°C? 1

Extrapolation (demonstrate understanding that $x = 0.6$ is beyond the range of the data on which the regression line was based)

1 | correct reasoning of some form

Question 22 (4 marks)

The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started. **1**

$A = 80 - 45$

$= 35 \text{ km}^2$ 1 c/w

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures. **2**

When $t = 14$ $A = 60$

$\therefore 60 = 80 - 45e^{14c}$

$e^{14c} = \frac{4}{9}$

$14c = \ln(4/9)$

$c = \frac{1}{14}\ln(4/9)$

$= -0.057923587$

$= -0.0579$ (3 sig fig)

2 | correct

1 | substitution and progress to find c

On 1st January 2020, the owners of the nature reserve announced a long term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan. **1**

As $t \to \infty$ $e^{-0.0579t} \to 0$

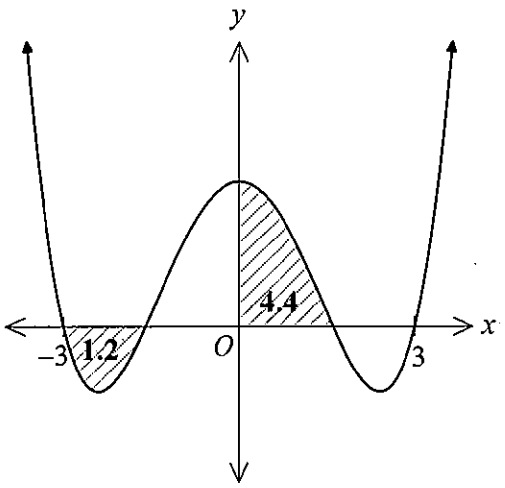
$\therefore A \to 80 \text{ km}^2 < 100 \text{ km}^2$

$\therefore$ Model will not achieve aim.

1 | correct reasoning of some form.

Question 23 (1 mark)

The graph of the function $y = f(x)$ below is symmetrical about the y-axis. The areas of the shaded regions are given.



Find $\int_{-3}^{3} f(x)\,dx$. **1**

$= 2 \times -1.2 + 2 \times 4.4$

$= 6.4$ 1 ✓/ω

Question 24 (3 marks)

Prove that $\dfrac{\sec x \tan x + \sec^2 x}{(\tan x + \sec x)^2 + 1} = \dfrac{1}{2}$. 3

LHS : $\dfrac{\sec x(\tan x + \sec x)}{\tan^2 x + \sec^2 x + 2\tan x \sec x + 1}$

$= \dfrac{\sec x(\tan x + \sec x)}{2\sec^2 x + 2\tan x \sec x}$; $\tan^2 x + 1 = \sec^2 x$

$= \dfrac{\sec x(\tan x + \sec x)}{2\sec x(\sec x + \tan x)}$

$= \dfrac{1}{2}$

$= RHS$

3	correct proof
2.	Factoring Numerator Expanding denominator Use of identity
1	Factoring numerator Expanding denominator } In a meaningful way

Question 25 (2 marks)

Determine the values of x for which the graph of $y = 3\cos\left(\frac{5x-1}{2}\right)$ crosses the x-axis in the interval $0 \le x \le \pi$. 2

$\cos\left(\frac{5x-1}{2}\right) = 0 \qquad 0 \le x \le \pi$

$\frac{5x-1}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \qquad -\frac{1}{2} \le \frac{5x-1}{2} \le \frac{5\pi-1}{2}$

$5x - 1 = \pi, 3\pi$

$5x = \pi + 1, 3\pi + 1$

$x = \frac{\pi+1}{5}, \frac{3\pi+1}{5}$

2	correct
1	one correct soln

Question 26 (2 marks)

Find the value of k, given that $\int_1^k \frac{2}{\sqrt{x}}\,dx = 8$. 2

$\int_1^k 2x^{-1/2}\,dx = 8$

$* \quad \left[2x^{1/2}\right]_1^k = 4$

$\sqrt{k} - 1 = 2$

$\sqrt{k} = 3$

$k = 9$

2	correct
1	correct integration *

Question 27 (6 marks)

Hen Solo made a profit of \$20 000 in its first year of trading with her free-range egg farm.
A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.
According to the model,

(a) show that the profit for Year 3 will be \$23 328. **1**

$T_1 = 20000$

$T_2 = 20000 \times 1.08$ 1 c/w

$T_3 = 20000 \times 1.08^2 = \$23328$

(b) find the first year when the yearly profit will exceed \$65 000. **3**

$T_n = 20000(1.08)^{n-1} > 65000$

$(1.08)^{n-1} > 3.25$

$(n-1)\ln(1.08) > \ln(3.25)$

$n - 1 > 15.31\ldots$

$n > 16.31\ldots$

∴ Profit will exceed \$65000 after 17 years

3	Correct
2	Correct inequality and progress to soln
1	Correct inequality (or equation)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest \$1000. **2**

$S_{20} = \dfrac{20000(1.08^{20} - 1)}{1.08 - 1}$

$= \$915000$

2	Correct
1	Correct sum formula with correct values; Error in calculation

Question 28 (7 marks)

A continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases}0 & x<1\\ \frac{2}{x^2} & 1\le x\le 2\\ 0 & x>2\end{cases}$$

(a) Show that $f(x)$ is a probability density function. 2

$\int_1^2 2x^{-2}\,dx = 2\left[-x^{-1}\right]_1^2$

$= 2\left\{-\frac{1}{2}+1\right\}$

$= 2\times\frac{1}{2}$

$= 1$

Since $f(x)=\frac{2}{x^2} > 0 \quad \forall\, x\in[1,2]$ and

$\int_1^2 f(x)\,dx = 1$, $f(x)$ is a pdf.

2 | Correct

1 | Just the integral without stating $f(x)\ge 0$

(b) Find the cumulative distribution function $F(x)$ of the random variable X. 2

$F(x) = 2\int_1^x x^{-2}\,dx$

$= 2\left[-x^{-1}\right]_1^x$

$= 2\left\{-\frac{1}{x}+1\right\}$

$\therefore F(x)=\begin{cases}0 & x<1\\ 2-\frac{2}{x} & 1\le x\le 2\\ 0 & x>2\end{cases}$

2 | correct

1 | Correct integral with boundaries

Question 28 continues on next page

Question 28 continued

(c) Show that if Q_1, Q_2, Q_3 are the quartiles of the distribution of X, then $\frac{1}{Q_1}, \frac{1}{Q_2}, \frac{1}{Q_3}$ are consecutive terms in an arithmetic sequence. **3**

$1 - \frac{2}{x} = \frac{1}{4}$ $\quad 2 - \frac{2}{x} = \frac{1}{2}$ $\quad 2 - \frac{2}{x} = \frac{3}{4}$

$-\frac{2}{x} = -\frac{7}{4}$ $\quad -\frac{2}{x} = -\frac{3}{2}$ $\quad -\frac{2}{x} = -\frac{5}{4}$

$\frac{x}{2} = \frac{4}{7}$ $\quad \frac{x}{2} = \frac{2}{3}$ $\quad \frac{x}{2} = \frac{4}{5}$

$x = \frac{8}{7}$ $\quad x = \frac{4}{3}$ $\quad x = \frac{8}{5}$

$\therefore Q_1 = 8/7$ $\quad Q_2 = 4/3$ $\quad Q_3 = 8/5$

$\frac{1}{Q_1} = \frac{7}{8}$ $\quad \frac{1}{Q_2} = \frac{3}{4}$ $\quad \frac{1}{Q_3} = 5/8$

$T_2 - T_1$

$3/4 - 7/8 = -1/8$ since $T_2 - T_1 = T_3 - T_2$

$T_3 \quad T_2$

$5/8 - 3/4 = -1/8$ $\frac{1}{Q_1}, \frac{1}{Q_2}, \frac{1}{Q_3}$ form

an arithmetic sequence

3	Correct
2	Correctly calculating Q_1, Q_2, Q_3 Progress towards showing an arithmetic sequence
1	Correct attempt at finding Q_1, Q_2, Q_3

Question 29 (5 marks)

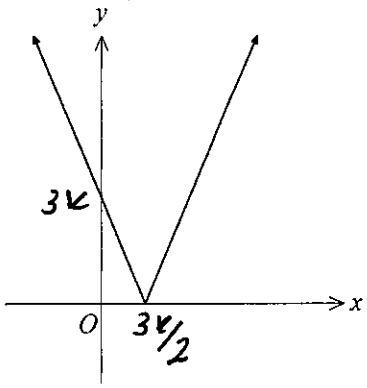


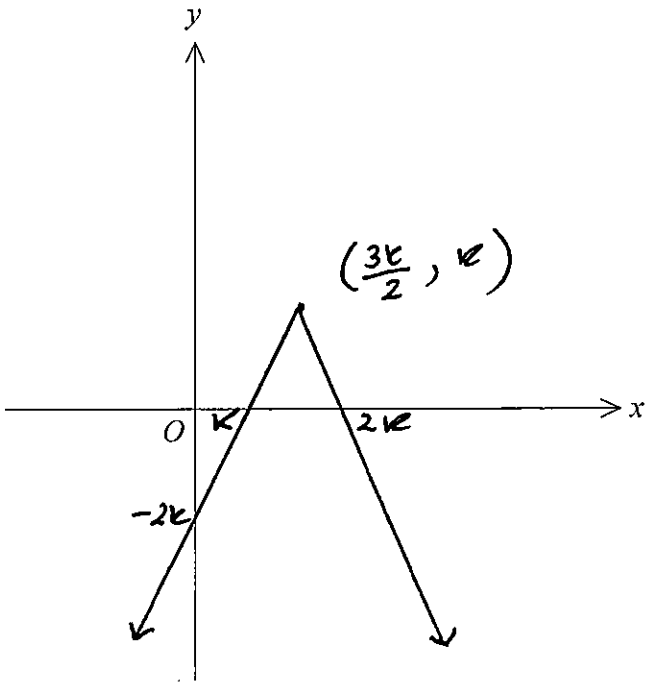
Figure 3

Figure 3 shows a sketch of the graph with equation $y = |2x - 3k|$ where k is a positive constant.

(a) Sketch the graph with equation $y = h(x)$ where $h(x) = k - |2x - 3k|$ stating **3**

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

3	Correct
2.	One piece of information missing OR 1 error
1	one of pieces of information Correct



y - intercept : $-2k$ Maximum : $\left(\frac{3k}{2}, k\right)$

x - intercepts $|2x - 3k| = k$

$2x - 3k = \pm k$

$2x = 3k \pm k$

$2x = 4k, 2k$

$x = 2k, k$

Question 29 continues on next page

Question 29 continued.

(b) Find, in terms of k, the coordinates of the minimum point of the graph with equation **2**

$$y = 3 - 5h\left(\frac{1}{2}x\right)$$

$h(\frac{1}{2}x) = k - |x - 3k|$ — 2 | Correct

$y = 3 - 5(k - |x - 3k|)$ — 1 | Correct attempt at finding y

$y = 3 - 5k + 5|x - 3k|$

Minimum point : $(3k, 3 - 5k)$

Question 30 (5 marks)

The velocity of a particle moving along the x-axis is given by

$$V = 10 - 10e^{-2t}$$

where t is the time in minutes and x is the displacement in metres.

(a) Show that the particle is initially at rest. **1**

when $t = 0$ $\quad V = 10 - 10e^{0}$

$= 0$ m/min

$\therefore$ Particle is initially at rest

1 r/w

(b) Show that the acceleration of the particle is always positive. **1**

$\frac{dV}{dt} = 20e^{-2t} > 0 \quad \forall\ t \geq 0$

$\therefore$ acceleration is always positive

1 r/w

(c) Explain why the particle is moving in a positive direction for all $t > 0$. **1**

For $t \geq 0$ $\quad V = 10 - 10e^{-2t}$

As $t \to \infty$ $\quad e^{-2t} \to 0$ $\therefore$ $V \to 10$

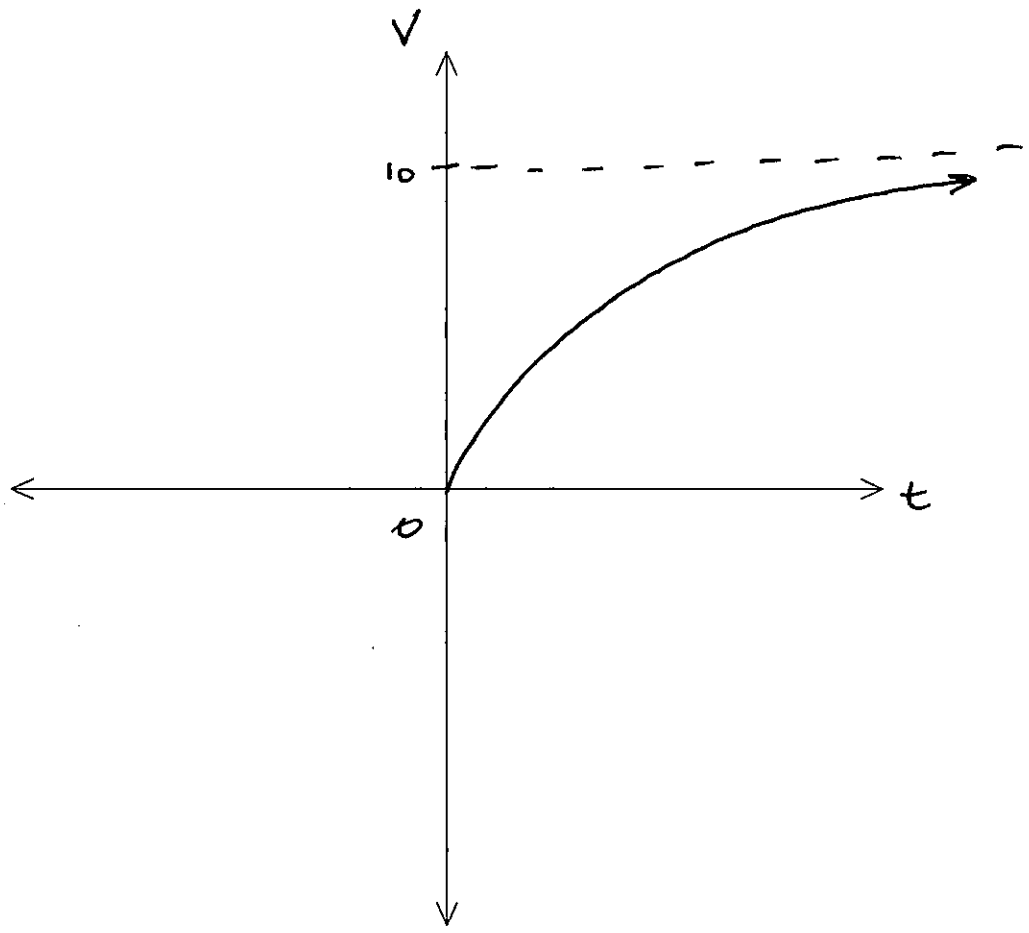
and $V \geq 0$ $\therefore$ particle always moves to the right

1 r/w

Question 30 continues on next page

Question 30 continued.

(d) Sketch the graph of the particle's velocity as a function of time. **2**



2	correct
1	Missing information

Question 31 (4 marks)

(a) Differentiate $3+\sin 2x$ **1**

$\frac{d}{dx}(3+\sin 2x) = 2\cos 2x$

1 ✓/ω

(b) Hence, or otherwise, find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 2x}{3+\sin 2x}\,dx$. **3**

$\int_{\pi/4}^{\pi/2} \frac{\cos 2x}{3+\sin 2x}\,dx = \frac{1}{2}\left[\ln|3+\sin 2x|\right]_{\pi/4}^{\pi/2}$

$= \frac{1}{2}\{\ln|3+\sin\pi| - \ln|3+\sin\tfrac{\pi}{2}|\}$

$= \frac{1}{2}\{\ln 3 - \ln 4\}$

$= \frac{1}{2}\ln(\tfrac{3}{4})$

3	correct
2	correct integral and substitution; error in calculation
1	• correct integral • constant incorrect or substitution

Question 32 (3 marks)

Given that $y=\dfrac{x-4}{2+\sqrt{x}}$, $x>0$ find the value of A such that $\dfrac{dy}{dx}=\dfrac{1}{A\sqrt{x}}$. **3**

$$\frac{dy}{dx}=\frac{(2+x^{1/2})-(x-4)(\frac{1}{2}x^{-1/2})}{(2+\sqrt{x})^2}$$

$$=\frac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+\frac{2}{\sqrt{x}}}{(2+\sqrt{x})^2}$$

$$=\frac{2+\frac{1}{2}\sqrt{x}+\frac{2}{\sqrt{x}}}{(2+\sqrt{x})^2}$$

$$=\frac{4\sqrt{x}+x+4}{2\sqrt{x}(2+\sqrt{x})^2}$$

$$=\frac{4\sqrt{x}+x+4}{2\sqrt{x}(4+x+4\sqrt{x})}$$

$$=\frac{1}{2\sqrt{x}}$$

$\therefore A=2$

3	Correct
2	Correct derivative and some progress to result
1	Correct derivative • mistake in deriv but correct sub into quotient rule

also

$$y=\frac{x-4}{2+\sqrt{x}}\times\frac{2-\sqrt{x}}{2-\sqrt{x}}$$

$$y=\frac{(x-4)(2-\sqrt{x})}{(4-x)}$$

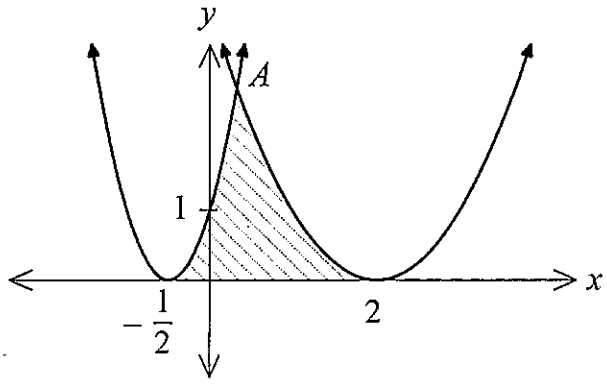
$$y=-(2-\sqrt{x})=\sqrt{x}+2$$

$$\therefore \frac{dy}{dx}=\frac{1}{2\sqrt{x}}$$

$\therefore \boxed{A=2}$

Question 33 (4 marks)

Two curves, $y=(2x+1)^2$ and $y=(x-2)^2$, intersect as shown in the diagram below.



(a) Show that A has x-coordinate $\frac{1}{3}$. **1**

1 r/w

no marks for $(x-2)^2=(2x+1)^2$ $\therefore x-2=2x+1$

$(2x+1)^2=(x-2)^2$ From the diagram A has x-coordinate 1/3

$4x^2+4x+1=x^2-4x+4$

$3x^2+8x-3=0$ $3x \times -1$, $x \times 3$

$(3x-1)(x+3)=0$

$\therefore x=1/3$ or $x=-3$

(b) The region bounded by $y=(2x+1)^2$, $y=(x-2)^2$ and the x-axis is shaded. **3**
Find the exact area of the shaded region.

$A=\int_{-1/2}^{1/3}(2x+1)^2\,dx+\int_{1/3}^{2}(x-2)^2\,dx$

$=\left[\frac{(2x+1)^3}{6}\right]_{-1/2}^{1/3}+\left[\frac{(x-2)^3}{3}\right]_{1/3}^{2}$

$=\left[\frac{125}{162}\right]-0+0-\left[\frac{-125}{81}\right]$

$=\frac{125}{54}$ units^2

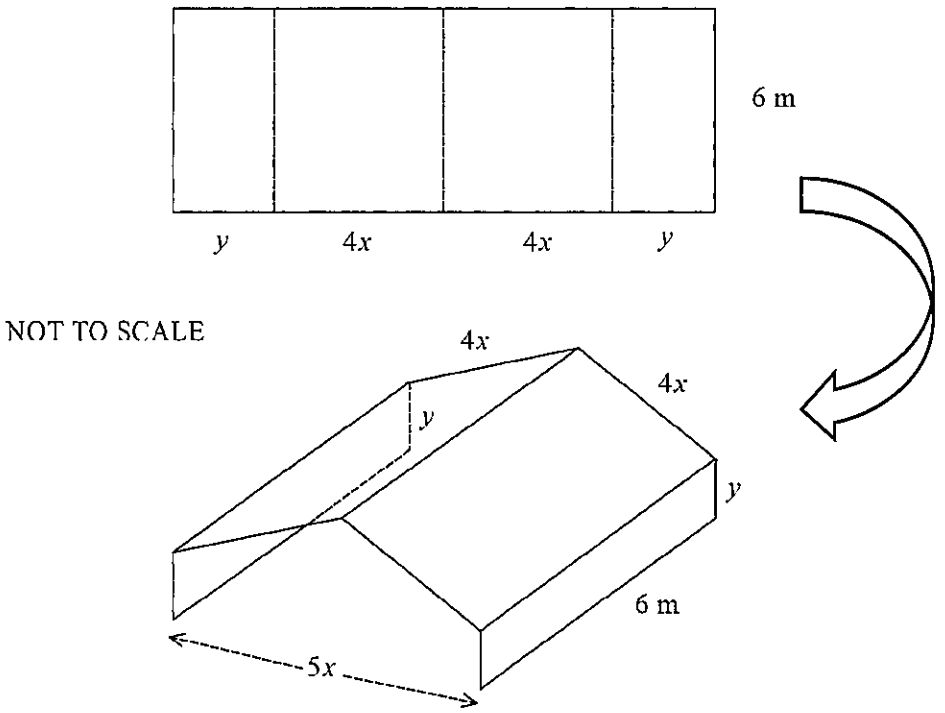
3 correct

2 correct integrals error in calculation

1 correct pieces corresponding to area

Question 34 (6 marks)

A rectangular canvas sheet is folded to form the cover of a tent with a pentagonal cross-section, as shown in the following diagrams.



The vertical heights on the sides of the tent are y metres and the slant lengths are $4x$ metres.
The horizontal distance between the vertical sides is $5x$ metres and the width of the tent is 6 metres.

(a) Show that $y = 10 - 4x$, given the area of the canvas sheet is 120 m^2. **1**

$6(2y + 8x) = 120$ ...1 mk

$2y + 8x = 20$

$2y = 20 - 8x$

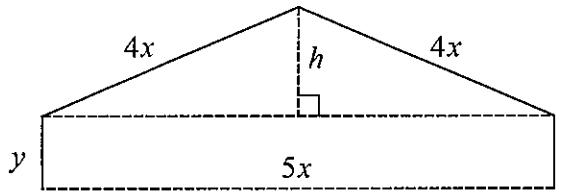
$y = 10 - 4x$

Question 34 continues on next page

Question 34 continued.

The following diagram shows the cross-section of the tent, where h is the height of the top of the tent from the vertical sides.

NOT TO SCALE



(b) Show that $h = \frac{\sqrt{39}x}{2}$. **2**

$(4x)^2 = (2.5x)^2 + h^2$

$h^2 = 16x^2 - 6.25x^2$

$h^2 = 9.75x^2$

$h^2 = \frac{39}{4}x^2$

$h = \frac{\sqrt{39}}{2}x$ As required

2	Correct
1	Correct Pythagorean statement, error in solution

Question 34 continues on next page

Question 34 continued.

(c) Find the exact value of x which maximises the volume of the tent. **3**

$A = 5x \times y + \frac{1}{2} h \times 5x$

$= 5x(10 - 4x) + \frac{5}{2} x \left(\frac{\sqrt{39}\, x}{2}\right)$

$= 50x - 20x^2 + \frac{5\sqrt{39}}{4} x^2$

$V = 6A$

$V = 300x - 120x^2 + \frac{15\sqrt{39}}{2} x^2$

$\frac{dV}{dx} = 300 - 240x + 15\sqrt{39}\, x$

$\frac{d^2V}{dx^2} < 0 \quad \therefore$ Volume will be a maximum.

When $\frac{dV}{dx} = 0 \qquad 240x - 15\sqrt{39}x = 300$

$x = \frac{300}{240 - 15\sqrt{39}} = \frac{20}{16 - \sqrt{39}}$

$x = \frac{320 + 20\sqrt{39}}{217}$

3	Exact value of x and show maximum
2	Correct Volume in terms of x and attempt to solve $\frac{dV}{dx} = 0$
1	Correct cross-sectional area or reasonable attempt of Volume

END OF PAPER

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